Induced Lorentz-Violating Chern-Simons Term in QED and Anomalous Contributions to Effective Action Expansions

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Abstract

I present a unified formulation of anomalous contributions in quantum field theories by calculating directly the effective action using the background field and covariant-derivative expansion technique. I use this method to determine uniquely the induced Chern-Simons term from the a Lorentz and CPT violating term in fermion QED Lagrangian. The outstanding ambiguity is resolved by properly taken into account the noncommutivity of A and ∂. The resulting vacuum polarization tensor acquires a non-Feynman diagram anomalous contribution which accounts for the discrepancy between the present calculation and the other calculations.

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Recently there has been increasing interest in the possibility of breaking Lorentz symmetry, by adding Lorentz-violating terms to the Lagrangian in the electromagnetic sector as well as the fermion sector [1–10]. Since no actual Lorentz violation has ever been observed, the upper limits of these Lorentz-violating coupling constants are severely restrained. The recent developments center on exploiting the unconventional physical consequences which may either enhance the probability of detecting such violations, or may set more stringent limits on the Lorentz-violating terms [1–5].

The electromagnetic Lorentz and CPT violating Chern-Simons term is $\frac{1}{2}\epsilon_{\mu\alpha\beta\nu}k^{\mu}F^{\alpha\beta}A^{\nu}$ [1,2], where k_μ is a constant 4-vector. It has been suggested [1] that astrophysical tests can provide a sensitive measure of the magnitude of this Lorentz-violating Chern-Simons term through the rotation of the plane of polarization of radiation over cosmological distances. Data from distant galaxies has been analyzed and reanalyzed for radiation from cosmological radio sources to a high degree of accuracy with no convincing evidence that such an effect exists [1,11].

The quantum electrodynamics (QED) Lagrangian of a single charged fermion field can be extended by inclusion of a CPT-odd Lorentz violating term [2],

$$\mathcal{L} = \bar{\psi}\Big[i\partial\!\!\!/ - m - \gamma_5 b\!\!\!/ - QA\!\!\!/(x)\Big]\psi \ , \tag{1}$$

where b_μ is a constant 4-vector and Q is the charge of the fermion. The effect of this violation can only be detected indirectly through precision measurements of CPT violation of physical properties of the fermion. The present knowledge of the upper limit on b_μ is far less stringent than on k_μ.

The interesting question is whether a Lorentz-violating CPT-odd term in the fermion sector can induce a finite Lorentz-violating Chern-Simons term in the electromagnetic field reminiscent of the $\pi^0 \to \gamma\gamma$ chiral anomaly [14,15]. In that case, without any unnatural fine-tuning or special constraints on model building, the stringent limit on k_μ would also imply a severe upper limit on b_μ so that the search for TCP violating effects from other high precision measurements may not be effective. Since such a Chern-Simons term is not gauge invariant, it may be logical to expect that a gauge invariant Lagrangian cannot induce a gauge non-invariant term in perturbation theory [5]. However, it has been suggested [2] that anomalous momentum surface integral terms of the type responsible for the existence of chiral anomaly may also contribute to a finite value of k_μ. Theories with anomaly are theories with one-loop anomalous contribution to the effective Lagrangian which violate the original gauge symmetry. While the recognition of anomalous contributions in quantum field theories has led to tremendous progress in particle physics, it remains a puzzle why quantum field theory can allow such undeterministic ambiguity [8]. In this paper, I propose how this puzzle of ambiguous anomalous contributions can be resolved. The solution may provide a necessary clue to the understanding the general problem of anomalies in quantum field theories.

The effective action is obtained by integrating out the fermion field in Eq. 1,

$$\Gamma_{eff} = -i\,Tr\,ln\Big[i\partial\!\!\!/ - QA\!\!\!/(x) - m - \gamma_5 b\!\!\!/\Big] \ , \tag{2}$$

where Tr stands for the trace in configuration space, spinor matrix space and internal symmetry space if it is included. The trace in configuration space is carried out by resolving $\delta^4(x-x') = \int \frac{d^4p}{(2\pi)^4} e^{-ip.(x-x')}$ [18], and the effective Lagrangian is given by

$$\mathcal{L}_{eff} = -i\int \frac{d^4p}{(2\pi)^4} tr\, ln\Big[\not p - m + i\not\partial - Q\not A(x) - \gamma_5 \not b\Big]. \tag{3}$$

Here p is the internal loop momentum and ∂ corresponds to the routing momentum of the external lines when the effective action is expanded into Feynman diagrams. It becomes important to note that the trace "tr" here does not apply to space-time. In any perturbation expansion, one partitions the argument of the logarithmic function into two parts, $\not p - m + X$ and Y, and expands the effective action using modified Hausdorf expansion,

$$tr\, ln\Big[\not p - m + X + Y\Big] = tr\, ln(\not p - m + X)(1 + \frac{1}{\not p - m + X} Y)$$
$$= tr\Big\{ln(\not p - m + X) + ln(1 + \frac{1}{\not p - m + X}Y) + \frac{1}{2}[ln(\not p - m + X), ln(1 + \frac{1}{\not p - m + X}Y)]$$
$$+ \text{ higher order commutators}\Big\}.$$

Irrespective of any gamma matrix dependence, if the space-time parts of the operators X and Y do not commute, as in the case of a Feynman graph expansion where $X = i\not\partial + \ldots$ and Y is a function of x, the commutator terms may not be zero. However, the loss of cyclic permutation invariance of the space-time operators can be compensated by an appropriate shift in the momentum integration variable p. As it is well-known from the axial anomaly, a shift of the variable of integration for a linear divergent integral contributes a nonzero finite change. For this reason, the commutator terms give a well-defined additional anomalous contribution to the effective Lagrangian beyond those calculated from the Feynman diagram reconstruction. If these commutators are taken into account, the results of any calculation should be independent of how expansions are carried out. We can avoid calculations of these commutators if the initial expansion is carried out by choosing X and Y so that the contribution from all commutator terms vanishes. A symmetrical choice is $X = 0$. For the effective action Eq. 2 where b_μ is constant, any combination keeping $\Pi_\mu = i\partial_\mu - QA_\mu$ as a single entity will satisfy this condition. They should all yield the same unique result with no ambiguity. The essence of the covariant-derivative-expansion is to develop a series of local effective Lagrangian in powers of Π_μ, rather than in powers of $i\partial_\mu$ and A_μ separately [17,19]. The choice $X = i\not\Pi$ and $Y = -\gamma_5\not b$ gives the simplest calculation. Expanding the effective action first in powers of $\gamma_5\not b$, and then in powers of $i\not\Pi$, we can calculate the Chern-Simons term,

$$\begin{aligned}\mathcal{L}_{eff} &= i\int \frac{d^4p}{(2\pi)^4} tr \frac{1}{\not p - m + \not\Pi}\gamma_5\not b + \ldots \\ &= -i\int \frac{d^4p}{(2\pi)^4} tr\, S\not\Pi S\not\Pi S\not\Pi S\gamma_5\not b + \ldots \\ &= 4b^\mu \Pi^\alpha \Pi^\beta \Pi^\kappa \epsilon_{\lambda\alpha\beta\nu}\int \frac{d^4p}{(2\pi)^4}\left[\frac{g^\nu_\kappa}{(p^2-m^2)^2} - \frac{4p_\kappa p^\nu}{(p^2-m^2)^3}\right]\end{aligned} \tag{4}$$

where $S = \frac{i}{\not p - m}$. The Lorentz and CPT violating Chern-Simons term is finite without any regularization, $k_\mu = \frac{1}{8\pi^2}Q^2 b_\mu$.

We now consider the general case of a massive fermion Lagrangian with $m \neq 0$,

$$\mathcal{L} = \bar{\psi}\Big[i\partial\!\!\!/ - m - Q A\!\!\!/(x) - \chi(x)\Big]\psi \ , \tag{5}$$

where $\chi(x)$ can include any combination of boson fields and any dependence on gamma matrices. The corresponding effective Lagrangian is

$$\mathcal{L}_{eff} = -i\int \frac{d^4p}{(2\pi)^4} tr\ \{ln\Big[p\!\!\!/ + \Pi\!\!\!/ - m - \chi(x)\Big] - ln(p\!\!\!/ - m)\}.$$

To isolate the anomalous part from the normal vector gauge invariant part of the effective action, I employ the transformation of Ref. [19], with the exception that I do not drop those momentum surface integral terms, which are the gauge-noninvariant contributions. I define the gauge invariant combination to be [19],

$$\begin{aligned} T &= e^{-\Pi\cdot\frac{\partial}{\partial p}}\, tr\, ln\Big[p\!\!\!/ + \Pi\!\!\!/ - m - \chi(x)\Big]e^{\Pi\cdot\frac{\partial}{\partial p}} \\ &= tr\, ln\Bigg[p\!\!\!/ - m - \frac{i}{2}Q\gamma^\mu \sum_{n=0}^{\infty}\frac{2(n+1)}{(n+2)!}\frac{1}{i^n}(D_{\mu_1}..D_{\mu_n}F_{\mu\nu})\frac{\partial^{n+1}}{\partial p_{\mu_1}..\partial p_{\mu_n}\partial p_\nu} \\ &\quad - \sum_{n=1}^{\infty}\frac{1}{n!}\frac{1}{i^n}(D_{\mu_1}..D_{\mu_n}\chi)\frac{\partial^n}{\partial p_{\mu_1}..\partial p_{\mu_n}} - \chi(x)\Bigg], \end{aligned}$$

where the covariant derivatives are defined by $D_\mu\phi = \frac{1}{i}[\Pi_\mu, \phi]$ and $[\Pi_\mu, \Pi_\nu] = [i\partial_\mu - QA_\mu, i\partial_\nu - QA_\nu] = -iQF_{\mu\nu}$. The gauge invariant part of the effective Lagrangian is,

$$\mathcal{L}^{inv}_{eff} = -i\int \frac{d^4p}{(2\pi)^4}[T - ln(p\!\!\!/ - m)]. \tag{6}$$

The gauge-noninvariant anomalous part is,

$$\mathcal{L}^{nonin}_{eff} = -i\int \frac{d^4p}{(2\pi)^4}\Big(e^{\Pi\cdot\frac{\partial}{\partial p}} - 1\Big)T = -i\int \frac{d^4p}{(2\pi)^4}\Big[\sum_{n=1}^{\infty}\frac{1}{n!}\Big(\Pi\cdot\frac{\partial}{\partial p}\Big)^n T\Big].$$

The summation term is the anomalous effective Lagrangian defined in this paper as the vector gauge noninvariant part. It would vanish completely if conventional gauge invariant regularization such as Pauli-Villars regularization or the dimensional regularization is used. The noncovariant part is directly tied to a total momentum derivative. Therefore the anomalous effective Lagrangian can be expressed as a surface integral. If the unadulterated dimensional regularization is used to evaluate the momentum integral, the anomalous effective Lagrangian vanishes. Independently, it is also clear that the power counting of the momentum of the integrand becomes sufficiently negative as the number of differentiations increases, so that the surface integral would vanish naturally for sufficiently high value of n in the sum, and only very few terms would contribute regardless of regularization.

Conventionally, regularization procedures are used to keep track of degrees of divergence for the purpose of renormalization. For finite terms, regularization should make no difference. However, for the anomalous effective Lagrangian, gauge invariant regularization schemes force all terms in the anomalous effective Lagrangian to be zero. For those finite terms, the role of regularization is no longer regulating divergent quantities but rather of imposing gauge invariance. If we take the stance that processes not absolutely forbidden can exist

and exact symmetry can be broken by radiative corrections, we should allow the possibility of not imposing gauge invariant regularization on nondivergent terms unless ambiguity and inconsistency occur.

For the calculation of the Lorentz violating Chern-Simons term, I choose $\chi = \gamma_5 \not{b}$ and select only the relevant Chern-Simons terms,

$$
\begin{aligned}
\mathcal{L}^{CS}_{eff} &= -i\int\frac{d^4p}{(2\pi)^4}\Pi\cdot\frac{\partial}{\partial p}tr\, ln\Big[\not{p} - m - \frac{i}{2}Q\gamma^\mu F_{\mu\nu}\frac{\partial}{\partial p_\nu} - \gamma_5\not{b}\Big] \\
&= i\int\frac{d^4p}{(2\pi)^4}tr\Pi\cdot\frac{\partial}{\partial p}\Big[\frac{1}{\not{p}-m}\frac{i}{2}Q\gamma^\mu F_{\mu\nu}\frac{\partial}{\partial p_\nu}\frac{1}{\not{p}-m}\gamma_5\not{b}\Big] \\
&= \frac{Q}{2}\int\frac{d^4p}{(2\pi)^4}[\Pi_\alpha F_{\mu\beta} + \Pi_\mu F_{\beta\alpha} + \Pi_\beta F_{\mu\alpha}]trS\gamma^\alpha S\gamma^\beta S\gamma^\mu S\gamma_5\not{b} \\
&= \frac{1}{16\pi^2}Q^2\epsilon_{\mu\alpha\beta\nu}b^\mu F^{\alpha\beta}A^\nu \ .
\end{aligned} \tag{7}
$$

I recover the same Chern-Simons term. It also becomes obvious that there is no anomalous contributions from higher order b_μ, since terms higher powers in b are necessarily accompanied by more convergent factor which renders the surface integrals zero [10].

Now I have traced that the ambiguity of the anomalous contribution is due to the inappropriate expansion of the "tr ln" function after the space-time trace has been evaluated. The problem of extra contributions due to nonvanishing commutators would not be present if the expansion can be performed before the space-time trace is carried out. However due to short distant singularity, we cannot carry out such an expansion directly on Eq. 2 without proper regularization. A very natural regularization has indeed been proposed to simplify the calculation of effective action covariant derivative expansion with internal symmetry [17]. The effective action in Eq. 2 is invariant under a finite momentum translation,

$$
\begin{aligned}
\Gamma_{eff} &= -i\,Tr\, e^{ip\cdot x}ln\Big[i\not{\partial} - Q\not{A}(x) - m - \gamma_5\not{b}\Big]e^{-ip\cdot x} \\
&= -i\,Tr\, ln\Big[\not{p} + \not{\Pi} - m - \gamma_5\not{b}\Big] \ .
\end{aligned} \tag{8}
$$

The arbitrary momentum p can then be averaged over the entire momentum space,

$$
\Gamma_{eff} = -i\frac{1}{\delta^4(0)}\int\frac{d^4p}{(2\pi)^4}\,Tr\, ln\Big[\not{p} + \not{\Pi} - m - \gamma_5\not{b}\Big] \ , \tag{9}
$$

where $\delta^4(0) = \int\frac{d^4p}{(2\pi)^4}$. Only after averaging can we fully use the cyclic permutation of the full trace and freely expand the "ln" function without any possible ambiguity due to extra commutator terms. The Chern-Simons actions from any possible expansion will all end up equivalent to the following form:

$$
\Gamma^{CS}_{eff} = -i\,\frac{1}{\delta^4(0)}\int\frac{d^4p}{(2\pi)^4}Tr\, S\not{\Pi}S\not{\Pi}S\not{\Pi}S\gamma_5\not{b} \ , \tag{10}
$$

which is the counterpart of Eq. 4. The calculations of taking trace of the gamma matrices and integrating the momentum are the same as before. The corresponding part of the calculation $\int d^4x\,\Pi^\alpha\Pi^\beta\Pi^\kappa\epsilon_{\lambda\alpha\beta\nu}$ is

$$
\begin{aligned}
\frac{1}{\delta^4(0)}\mathcal{T}r\Pi^\alpha\Pi^\beta\Pi^\kappa\epsilon_{\lambda\alpha\beta\nu} &= \int d^4x\frac{1}{\delta^4(0)}\langle x|\Pi^\alpha\Pi^\beta\Pi^\kappa|x\rangle\epsilon_{\lambda\alpha\beta\nu} \\
&= -\frac{i}{2}Q\int d^4x\frac{1}{\delta^4(0)}\langle x|F^{\alpha\beta}(i\partial^\kappa - QA^\kappa)|x\rangle\epsilon_{\lambda\alpha\beta\nu} \\
&= \int d^4x\frac{i}{2}Q^2F^{\alpha\beta}A^\kappa\epsilon_{\lambda\alpha\beta\nu}\ , \qquad (11)
\end{aligned}
$$

where I have used the identities $\langle x|i\partial^\kappa|x\rangle = 0$ and $\langle x|x\rangle = \delta^4(0)$. Therefore I arrive at the same Chern-Simons effective Lagrangian again.

The entanglement of expanding two noncommuting operators may also be bypassed if we differentiate the expression with respect to m first,

$$
\frac{\partial}{\partial m}tr\,ln\Big[\not p - m + X + Y\Big] = -tr\frac{1}{\not p - m + X + Y} \qquad (12)
$$

and then the identity $\frac{1}{A+B} = \frac{1}{A} - \frac{1}{A}B\frac{1}{A+B}$ may be repeatedly used for further expansion. The final result should be independent of how the expansion is carried out. I rewrite the effective Lagrangian Eq. 3:

$$
\mathcal{L}_{eff} = -i\int\frac{d^4p}{(2\pi)^4}\int_m^\infty dm\,tr\,\frac{1}{\not p - m + \not\Pi - \gamma_5\not b}\ , \qquad (13)
$$

and expand it in powers of $\not\Pi$ and $\gamma_5\not b$ to obtain the Chern-Simons Lagrangian,

$$
\begin{aligned}
\mathcal{L}^{CS}_{eff} = &-\int\frac{d^4p}{(2\pi)^4}\int_m^\infty dm\,tr\Big[S\not\Pi S\not\Pi S\not\Pi S\gamma_5\not b S + S\not\Pi S\not\Pi S\gamma_5\not b S\not\Pi S \\
&+ S\not\Pi S\gamma_5\not b S\not\Pi S\not\Pi S + S\gamma_5\not b S\not\Pi S\not\Pi S\not\Pi S\Big] \qquad (14) \\
= &\ 4i\,\epsilon_{\mu\alpha\beta\nu}b^\mu\Pi^\alpha\Pi^\beta\Pi^\nu\int\frac{d^4p}{(2\pi)^4}\frac{m^2}{(p^2-m^2)^3} = \frac{i}{16\pi^2}Q^2F^{\alpha\beta}A^\kappa\epsilon_{\lambda\alpha\beta\nu}.
\end{aligned}
$$

The perturabtion expansion in powers of A can now be carried out easily from the expression in Eq.13 to yield the quadratic term in A,

$$
\mathcal{L}^{A^2}_{eff} = i\,Q^2\int\frac{d^4p}{(2\pi)^4}\int_m^\infty dM\,tr\,\frac{1}{\not p - M + i\not\partial - \gamma_5\not b}\not A(x)\frac{1}{\not p - M + i\not\partial - \gamma_5\not b}\not A(x)\frac{1}{\not p - M + i\not\partial - \gamma_5\not b} \qquad (15)
$$

$$
= -\int\frac{d^4p}{(2\pi)^4}\frac{i}{2}\,Q^2A_\mu(p)A_\nu(-p)\Pi^{\mu\nu}(p). \qquad (16)
$$

The vacuum polarization tensor

$$
\begin{aligned}
\Pi^{\mu\nu}(p) = &\int\frac{d^4k}{(2\pi)^4}tr\Bigg\{\gamma^\mu\frac{i}{\not k - m - \gamma_5\not b}\gamma^\nu\frac{i}{\not k + \not p - m - \gamma_5\not b} \qquad (17) \\
&+ i\int_m^\infty dM\Big[\gamma^\mu\Big(\frac{i}{\not k - M - \gamma_5\not b}\Big)^2\gamma^\nu\frac{i}{\not k + \not p - M - \gamma_5\not b} - \gamma^\mu\Big(\frac{i}{\not k - \not p - M - \gamma_5\not b}\Big)^2\gamma^\nu\frac{i}{\not k - M - \gamma_5\not b}\Big]\Bigg\}
\end{aligned}
$$

has been rewritten into an expression in terms of the standard one-loop vacuum polarization Feynman amplitude with b_μ-exact fermion propagators [6,7,10] plus a b_μ-exact correction

from the noncommutativity of the operators A and ∂. This correction, even though expressed in a closed form via a mass integration representation as the difference of two terms differed by a shift of integration variable ($k \to k - p$), is nonetheless nonzero. It gives precisely the additional contribution $-\frac{1}{32\pi^2}Q^2 b_\mu$ to k_μ to combine with the previous calculation [6,7,10] of the first term to yield $k_\mu = \frac{3}{32\pi^2}Q^2 b_\mu - \frac{1}{32\pi^2}Q^2 b_\mu = \frac{1}{8\pi^2}Q^2 b_\mu$. Therefore we have shown that there is no ambiquity in the calculation of k_μ as long as the noncommutativity of the operators A and ∂ is taken into account. Whether the calculation is done nonperturbatively in b_μ or not makes no difference.

The answer to the question whether a Lorentz-violating CPT-odd term in the fermion sector can induce a finite Lorentz-violating Chern-Simons term in the electromagnetic field hinges on the answer to a much more general question: must an apparently finite nongauge invariant effective Lagrangian term from radiative correction be forced to vanish by imposing a gauge invariant regularization on it? The authors of Ref. [6] remind us that an alternative must be allowed in order for the axial anomaly to exist. However, within this new domain, the conventional wisdom in reconstructing the effective Lagrangian from on the Feynman diagram expansion becomes ambiguous. There is an intrinsic quantum incompatibility between the power expansion of the photon fields and the derivative (small momentum) expansion. The subtlety requires a correction to the standard perturbation theory expansion similar in spirit to the modification of the T-product to the T^*-product. The calculation can be made much simpler by using the back ground field formulation of the effective action covariant-derivative-expansion. While the covariant-derivative-expansion preserves gauge symmetry when the symmetry is exact, it also serves as a regulator when gauge symmetry is forced to be broken by radiative corrections. I have used different approaches to explore various aspects of the anomalous contribution to the effective Lagrangian. The calculations from various covariant derivative expansions all converge to a unique Chern-Simons term $\frac{1}{16\pi^2}Q^2\epsilon_{\mu\alpha\beta\nu}b^\mu F^{\alpha\beta}A^\nu$. I have also derived the complete expression for the vacuum polarization tensor to account for the discrepancy between my calculation and the previous calculations [6,7]. Since the effect of the Chern-Simons term has not been detected, the minor difference in its numerical coefficient may not appear to have much significance. However in resolving the ambiguity in its calculation, I have demonstrated how the modern background field calculation and the effective action expansion method can be used to understand the complexity of quantum field theory calculations. This new approach may shed more light on the anomalous contributions to the quantum field theories.

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